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# STATISTICAL ANALYSIS OF EARTHQUAKE OCCURRENCES IN THE VICINITY OF HONG KONG

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## Abstract

Earthquake occurrences in the vicinity of Hong Kong were analysed statistically in terms of magnitude and intensity. Extreme value theory was used to determine the return periods of earthquakes greater than or equal to selected magnitudes. A monte Carlo simulation employing results of the extreme value analysis and the distribution of epicentre locations as well as a simple intensity model was used to generate the statistical distribution of intensity in Hong Kong. The results were compared with those obtained from an empirical relationship.

## 1. Introduction

Hong Kong is situated on the coast of south China where a number of geological fractures exist (Lin, 1980) (Fig. 1). Past records indicate that those fractures in the vicinity of Hong Kong are far less active seismologically than the major seismic belts on earth. There are on the average about three earthquakes felt by Hong Kong residents each year. Most of these earth tremors occur in an area around Heyuan, a town about 16 km north-northeast of Hong Kong, along a fault running from northeast to southwest. The remainder originates from either the Yangqiang area or from the Shantou area. Since 1067 A.D., there have been 8 earthquakes of magnitude 6 or above known to occur within 320 km of Hong Kong but none of them were closer than 150 km (中国科学院, 1974). Of these eight, intensity records in Hong Kong are available only for the last three. They are VI, V and V on the modified Mercalli scale recorded respectively in the 1918 Shantou earthquake magnitude (7.3 $M_s$), the 1962-Heyuan earthquake (magnitude 6.1 $M_s$) and the 1969 Yangjiang earthquake (magnitude 6.4 $M_s$).

For a meaningful assessment of the probability of occurrence of damaging earth tremors in Hong Kong, it is necessary to determine the time distribution of the intensity of the earth tremors felt in Hong Kong. An attempt to achieve this objective using a combination of extreme value analysis and Monte Carlo simulation is made. Results are compared with those derived from a simple empirical analysis of the distribution of recorded intensity.

## 2. The Data

The area of interest is chosen to be 320 km around Hong Kong (Fig. 1). The reasons for this choice are threefold. Firstly, at least one magnitude 7 earthquake has occurred historically in this area which gives rise to damage in Hong Kong with an intensity VI on the modified Mercalli scale. Secondly, records in Hong Kong show that over 95% of tremors felt in Hong Kong are due to earthquakes within this area. Thirdly, complete records of earthquakes of magnitude greater than or equal to 3 in this area are

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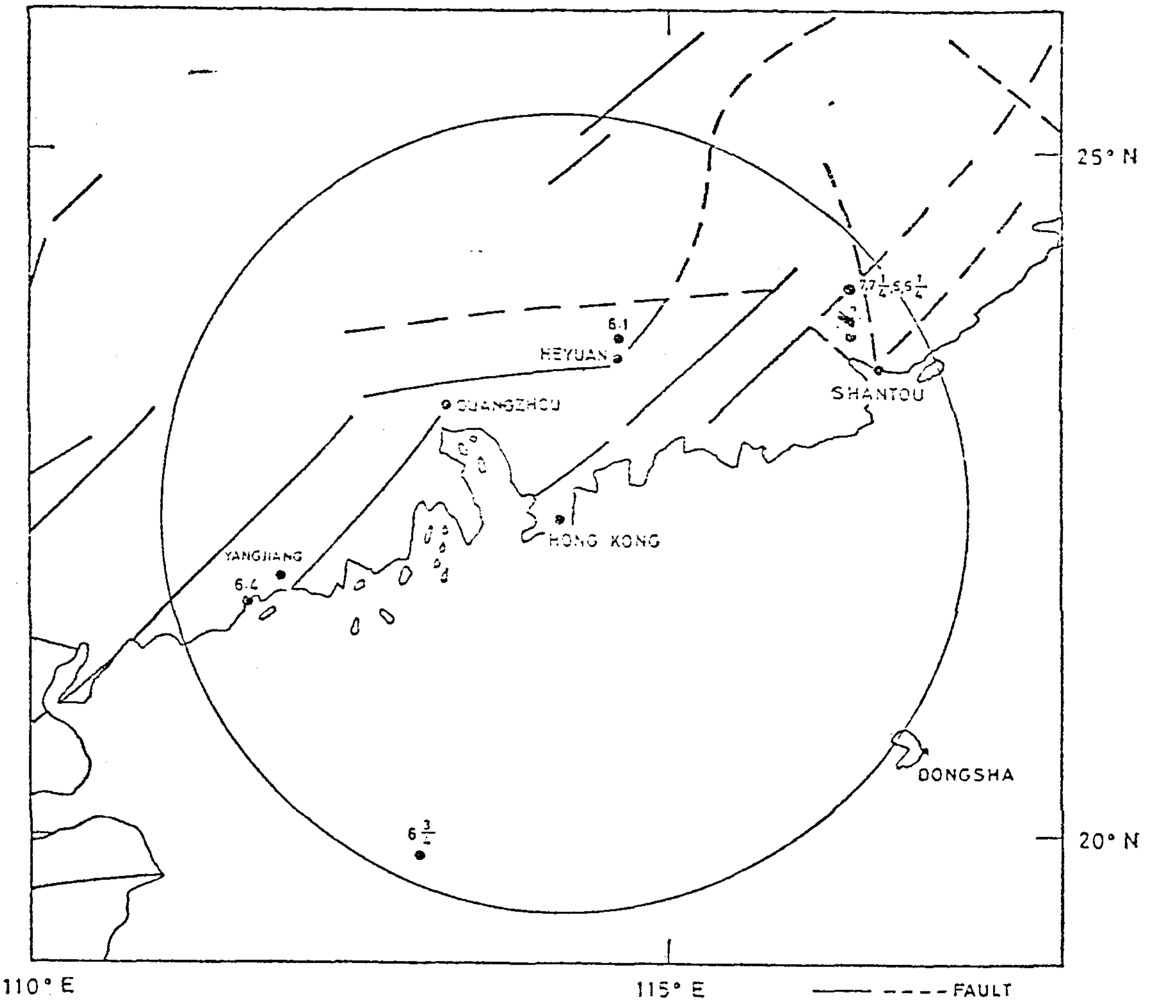

Fig. 1 Faults in the vicinity of Hong Kong (After Lin 1980) and earthquakes with $Ms \geq 6$ within 320 km of Hong Kong since 1067 A.D. (中国科学院 1974).

available from the State Seismological Bureau of China (1974, 1977) and the Guangdong Seismological Bureau (1977) for the period 1960-77, thus enabling further analysis to be made.

Intensity records are available in Hong Kong for 51 years from 1918 to 1940 and from 1954 to present. These records were in the modified Mercalli scale. The completeness of these records, particularly those in the early years and of lower intensities, is subject to some degree of uncertainty since they rely on the readiness of the public to report tremors.

## 3. Extreme Value Analysis of Earthquake Magnitudes

Let $N(M)$ be the number of earthquake per year with magnitudes greater than or equal to $M$. $N(M)$ is related to $M$ by the usual Gutenberg-Richter relationship (Gutenberg and Richter 1954):

$$\log N(M) = A - bM \qquad (1)$$

The cumulative probability of an earthquake having a magnitude less than or equal to $M$ is given by

$$F(M) = 1 - \frac{\int_{M}^{\infty} dN}{\int_{0}^{\infty} dN} = 1 - e^{-BM} \tag{2}$$

where $B = b\ln 10$.

Following Chen and Lin (1973), the probability of occurrence of $k$ earthquakes $P(k)$ is assumed to follow the Poisson distribution, i.e.

$$P(K) = \frac{a^{K} e^{-a}}{K!} \tag{3}$$

where $a$ is the average number of earthquakes occurred each year. The probability $G(M)$ that the maximum magnitude of earthquakes occurring in a year is less than or equal to $M$ is given by

$$G(M) = \sum_{k=0}^{\infty} P(k) [F(M)]^{k} \tag{4}$$

Substituting (2) and (3),

$$G(M) = e^{-e^{-B(M-u)}}$$

where $a = e^{Bu}$ and $A = bu$. (5)

Eq. (5) is identical in form to that given by the Gumbel $I$ distribution. To solve for $B$ and $u$, the earthquakes with the highest magnitude in each year in the period 1960-77 are selected and arranged in an ascending sequence $M_j$ for $j = 1,2,\ldots, 18$ as shown in the following table.

**Table 1. Magnitude data used in extreme value analysis**

| $J$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $M_j$ | 3.5 | 3.7 | 3.7 | 3.8 | 3.9 | 4.0 | 4.1 | 4.1 | 4.1 | 4.1 | 4.2 | 4.3 | 4.3 | 4.4 | 4.4 | 4.8 | 5.5 | 5.8 |

It can be shown (Gumbel, 1958) that the expected value of $G(M_j)$ is $j/(m+1)$ where $m = 18$ is the number of years of analysis. Substituting this value into Eq. (5), we have

$$-\ln\left[-\ln\left(\frac{j}{m+1}\right)\right] = B(M_j - u) \tag{6}$$

for $j = 1,2,\ldots, 18$. Eq. (6) is solved using the least square method giving

$B = 1.905$
$u = 3.988$

The correlation coefficient is 0.96 which is significant at better than the 0.1% level (Fig. 2).

The average number of earthquakes occurring each year having a magnitude greater than or equal to $M$ is

$$N(M) = a[1 - F(M)] \tag{7}$$

Hence the return period of an earthquake with a magnitude greater than or equal to $M$ is

$$T(M) = 1/N(M) = e^{B(M-u)} \tag{8}$$

Table 2 shows the return periods of some selected magnitudes as well as the magnitude with a 1000 year return period.

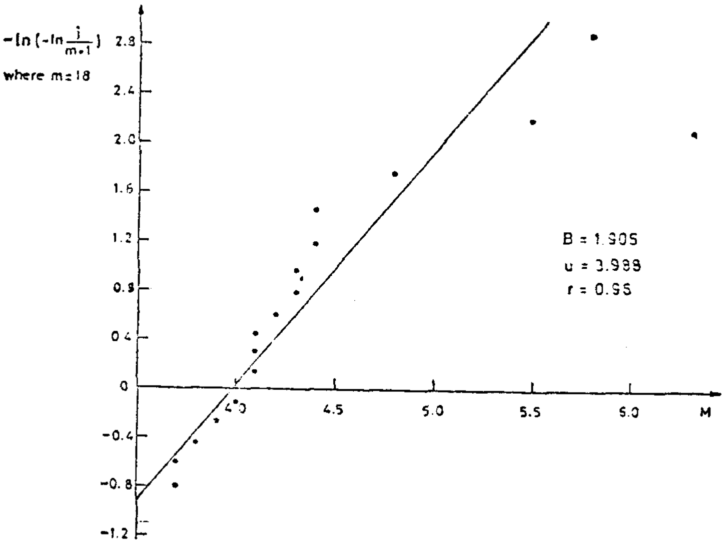

Fig. 2 Extreme value distribution of magnitudes within 320 km of Hong Kong (1960-1977).

It is generally recognised (Cosentino et al., 1977; Lomnitz-Adler and Lomnitz, 1979; Berrill and Davis, 1980; Makjanic, 1980; Howell, 1981) that there is a possible upper limit on the magnitude of an earthquake that can occur in a particular region. Chen and Lin (1973) modified Eq. (2) by replacing the upper limit of the definite integrals by a maximum magnitude $M2$. Eq. (6) and (8) thus became

$$-\ln\left[-\ln\left(\frac{j}{m+1}\right)\right] = B(M_j - u) - \ln\left[1 - e^{-B(M2 - M_j)}\right] \tag{9}$$

$$T(M) = \frac{e^{B(M-u)}}{1 - e^{-B(M2-M)}} \tag{10}$$

Eq. (9) is non-linear but can be solved by iteration using a modified Gauss-Newton method proposed by Marquardt (1963). Unfortunately, the data as shown in Table 1 are in the low magnitude region. They are not sufficient to determine a realistic value for the upper limit $M2$. In order to take into account the effect of the upper limit, the value of $M2 = 7.73$ determined by Chen and Lin (1973) using earthquake data collected in the coastal areas of southeast China is adopted. Using this value, return periods of magnitudes are corrected as shown in Table 2. The corrections are significant only for magnitudes 7 or above.

The probability $P_r$ that an indicated value of magnitude $M$ with a return period $T$ will be reached or exceeded at least once in a period $r$ is given by (Hershfield 1973)

$$P_r = 1 - \left[1 - (1/T)\right]^r \tag{11}$$

Values of $P_r$ for $r$=50 years are also tabulated in Table 2.

**Table 2. Return periods of earthquakes with magnitudes greater than or equal to $M$ within 320 km of Hong Kong**

| $M$ | Return Period $T$ ($M$) by (8) (years) | Return Period $Tl$ ($M$) by (10), corrected for upper limit $M2$ (years) | Probability of occurring at east once in 50 years based on the corrected return period (%) |
|---|---|---|---|
| 5.5 | 46 | 18 | 94 |
| 6 | 18 | 48 | 65 |
| 6.5 | 120 | 133 | 31 |
| 7 | 310 | 413 | 11 |
| 7.3 | 550 | 1000 | 5 |
| 7.5 | 804 | 2267 | 2 |

From Table 2, it is observed that the return period of a damaging earthquake, i.e. one with magnitude greater than or equal to 6.5, is over 100 years. The odds of one occurring during the life span of an ordinary building, which is typically taken to be 50 years by the Hong Kong building authority, is roughly one in three. In order to determine the effect of such magnitude distribution on buildings in Hong Kong, the time distribution of intensity and hence ground acceleration is next examined.

## 4. A Monte Carlo Simulation of Earthquake Intensity in Hong Kong

The distribution of intensity of earth tremors felt in Hong Kong is simulated using statistics of instrumentally recorded data by the Monte Carlo method (Hammersley and Handscomb, 1964). The parameters describing an earthquake are treated as independent random variables. The parameters that will affect the intensity of a tremor are $M$, the local magnitude, and $D$, the distance of the earthquake from Hong Kong. As a first approximation, it is assumed that intensity is independent of the azimuth of the earthquake from Hong Kong.

The cumulative frequency distribution for $M$ greater than or equal to 3 is obtained using Eq. (2) and the results of the extreme value analysis as follows:

$$F(M) = 1 - \frac{e^{-BM}}{e^{-3B}} \tag{12}$$

where $B = 1.905$ and $M$ has an upper limit of 8 in accordance with the value of $M$ 2 found in this area.

The cumulative frequency distribution of $D$ is extracted from the 39 historical earthquakes in the period 1960—77. As noted previously, a large proportion of the earthquakes is located between 120 and 160 km from Hong Kong in the Heyuan area. No common analytical distribution is found to fit the data to an acceptable confidence level. Thus the unsmoothed distribution as shown in Fig. 3 is used in the simulation.

A simple intensity model is adopted in the simulation as follows. Following Adams and Lowry (1968) the epicentral intensity of an earthquake with magnitude $M$ and depth $h$ is

$$I_{\max} = 1.5M - 3.5 \log h + 3.0 \tag{13}$$

In the present data set, the depths of earthquakes are not completely available to derive a cumulative distribution of $h$ to be used in the simulation. However, Huang and Xia (1982) observed that the depths of earthquakes in the coastal areas of south China have little variations, ranging from 5 to 20 km with most of them around 10 km, $h$ is therefore assumed to be a constant of 10 km in the present study.

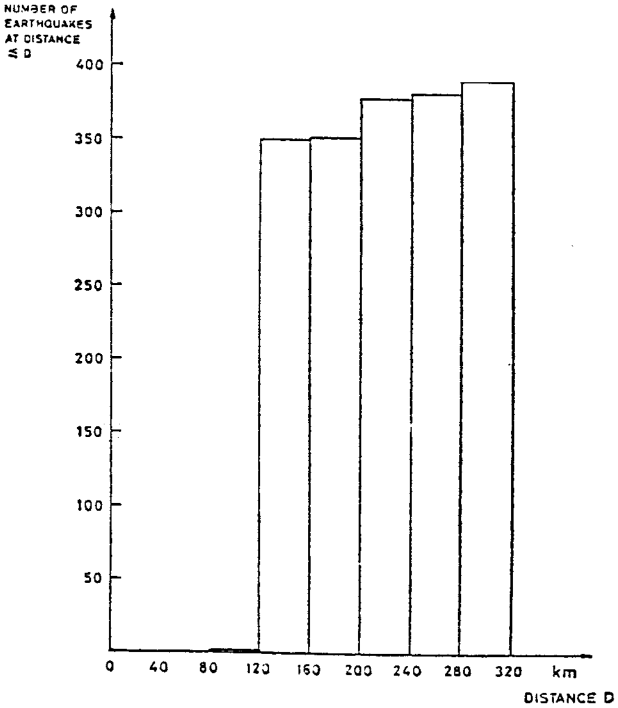

Fig. 3 Cumulative distribution of distances of earthquakes from Hong Kong.

The attenuation of intensity with distance is obtained empirically from an isoseismal map of the 1918 Shantou earthquake published by the State Seismological Bureau of China (1979) which is the only major earthquake in the intensity data set in the period 1918-40 and 1954-81. 24 values of distances of isoseismal lines from the epicentre on an 8-point compass are measured. It is found that they fit an empirical relationship of the form

$$I - 1 = (I_{\max} - 1)e^{-kD} \tag{14}$$

Using the method of least squares, it is found that the attenuation coefficient k is 0.002982 with a correlation coefficient of 0.97. This is significant at better than the 0.1% level (Fig. 4). Eq. (14) is adopted in the intensity model. Following the usual practice to assign an intensity value to an event only when that value is reached or exceeded, the integral part of the intensity values obtained by this model is used to construct probability distributions in the subsequent simulation.

The simulation procedure is as follows. An occurrence for each parameter is simulated by generating a random number, the range of which carries a one to one relationship with the cumulative probability distribution of each parameter. A set of simulated values for the two parameters constitutes a simulated earthquake. The intensity at Hong Kong associated with each earthquake is calculated using the intensity model. The simulation procedure is repeated enough times so that the effect distribution generated becomes stable.

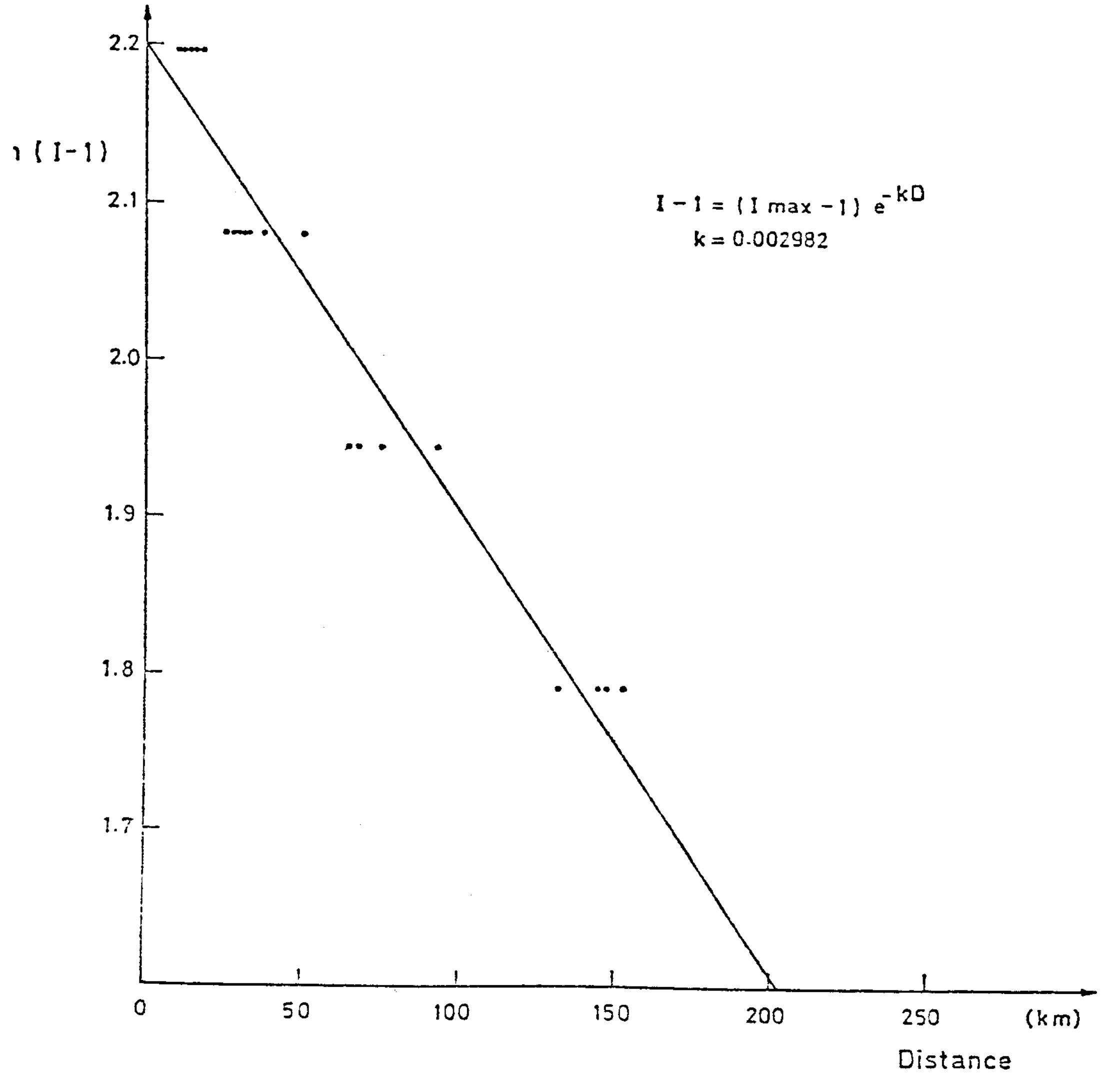

Fig. 4 Variation of intensity with distance in the 1918 Shantou earthquake.

The average number of earthquakes that occur each year is given by the extreme value analysis as

$$a = e^{Bu} = 1992$$

In order to simulate the return periods of events of the order of several hundred years, 1,200,000 simulated earthquakes have been used in this study corresponding to a period of around 600 years.

The result of Monte Carlo simulation shown in Fig. 5 is the cumulative probability effect distribution of *I* given an earthquake occurrence. The time element is incorporated by considering the probability *Pl (j)* of *j* earthquakes of magnitude 3 or higher occurring in each year. *Pl (j)* is obtained empirically using historical data. Let *F (I)* be the cumulative probability effect distribution of intensity *I* given an earthquake occurrence. *G (I)*, the cumulative probability that the intensities of all tremors felt in Hong Kong are less than or equal to I in an interval of one year, is given by

$$G(I) = \sum_{j=0}^{\infty} pl(j) F(I)^j \qquad (15)$$

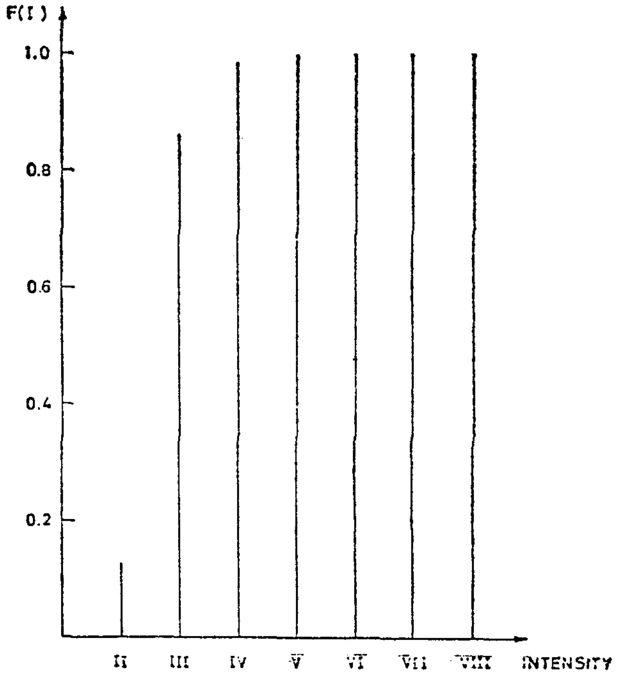

Fig. 5 Cumulative probability effect distribution *F(I)* of intensity by Monte Carlo method.

The return period *T (I)* for an earth tremor with intensity *I* or greater is then

$$T(I) = \frac{1}{1 - G(I)} \qquad (16)$$

Based on 1.2 million simulated earthquakes, the cumulative probability distribution *G (I)* for the time interval of one year of the intensity of an earth tremor in Hong Kong are derived and shown in Fig. 6.

## 5. Empirical Analysis of Earthquake Intensity in Hong Kong

The intensity of a felt earth tremor as given by the modified Mercalli scale is subjectively determined from descriptive reports or field investigations of earth movements. While they are not obtained instrumentally, examination of a large collection of description, if available, will enable the seismologist to assign an unambiguous intensity to the event. Provided that every tremor is reported, representative cumulative distribution of intensity can be drawn-up. Table 3 shows the number of earth tremors felt in Hong Kong as reported in the periods 1918—40 and 1954—81 that have intensities greater than or equal to the indicated value *I*. The number of events reported having intensity between I and II is expected to be lower than that actually occurred because of the weakness of the tremor. Thus they were not included in Table 3 as part of the data set for subsequent examination.

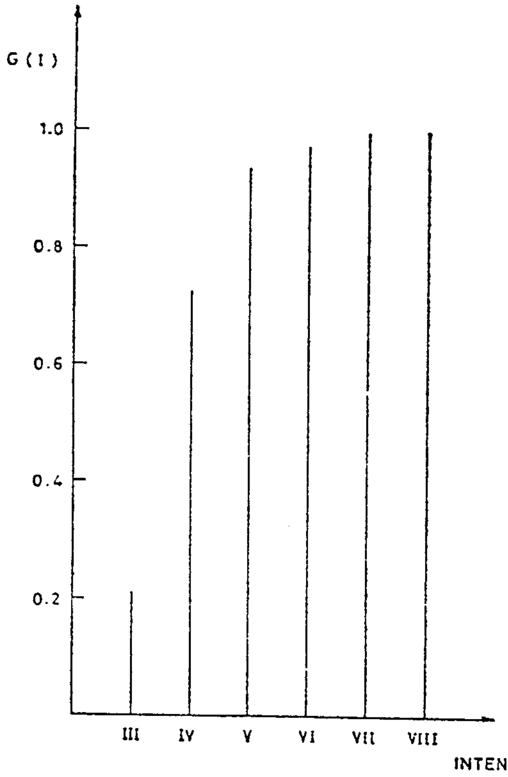

Fig. 6 Cumulative probability distribution *G(I)* of intensity in Hong Kong for the time interval of one year.

**Table 3.** **Cumulative distribution of intensity in Hong Kong (1918-40, 1954-81)**

| Intensity I | Number of events greater than or equal to I in 51 years |
|---|---|
| II | 80 |
| III | 32 |
| IV | 10 |
| V | 3 |
| VI | 1 |

It was found that this distribution fitted an empirical relation of a form similar to Eq. (1):

$$\log N(I) = A - bI \qquad (17)$$

where $N(I)$ is the number of events per year greater than or equal to intensity $I$. A and b are empirical constants found by the least square method to be

$$A = 1.203$$

$$b = 0.483$$

The correlation coefficient is 0.99 which is significant at better than the 0.1% level (Fig. 7). The return period T $(I)$ is given by $1/N(I)$.

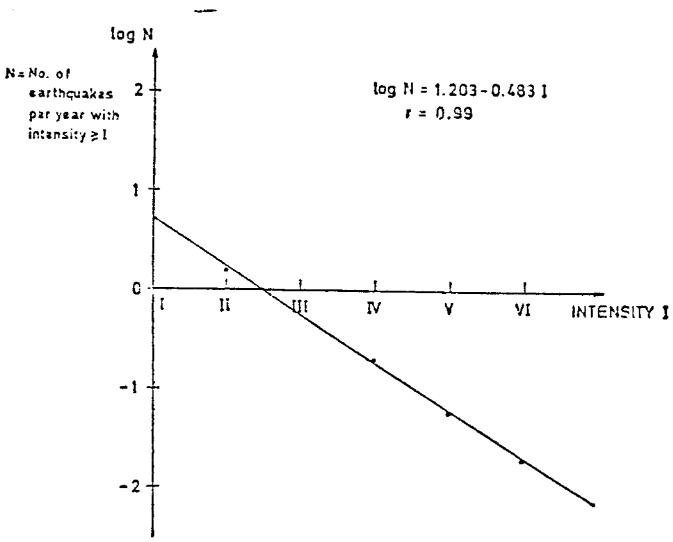

Fig. 7 Cumulative distribution of intensity in Hong Kong (1918-1940, 1954-1981).

## 6. Results and Discussions

Table 4 shows the return periods $T(I)$ of selected intensity as given by the Monte Carlo simulation and the empirical analysis respectively.

**Table 4. Return periods of earth tremors with intensity I or greater**

| Intensity $I$ | Return period by Monte Carlo simulation | Return period by empirical analysis |
|---|---|---|
| III | 1.3 | 1.8 |
| IV | 3.6 | 5.3 |
| V | 16 | 16 |
| VI | 33 | 50 |
| VII | 463 | 151 |
| VIII | — | 458 |

It is observed that the results are generally in mutual agreement for intensities VI or lower indicating that the Monte Carlo approach is valid. The return period of intensity VII estimated by the Monte Carlo simulation is larger than that obtained by empirical analysis. This is reasonable since the latter does not take into account the existence of an upper limit of magnitude in this region. The effect of this upper limit is to increase the return periods of high intensity events. The Monte Carlo simulation on the other hand has taken into account this upper limit. It should be noted that in repeated runs of

200,000 events, the estimated return periods of intensity VII range from 331 years to 661 years. This variation arises since the return period is comparable to the number of years of simulated records. It does however demonstrate that the occurrence of an intensity VII tremor in Hong Kong rather infrequent.

Another observation is that the return periods of intensities VIII or higher are not determined by the simulation because not a single event is generated out of the 1.2 million simulations.

The building code in Hong Kong requires buildings built after 1959 to withstand winds from tropical cyclones. It has been shown by the Hong Kong building authority that such buildings can withstand a horizontal ground acceleration of up to $0.07g$ with little or no damage. Using Richter's relation (Richter, 1958)

$$\log \alpha = I/3 - 1/2 - \log 981 \tag{18}$$

where $a$ is the ground acceleration in units of $g$, the acceleration due to gravity, the corresponding threshold intensity for consideration is VII. Assuming again that a building is built at the present time and has a life span of 50 years, the average number of events estimated to occur during its life span using return periods given by the Monte Carlo method is tabulated in Table 5. The probability that a particular intensity will be reached or exceeded at least once during this period is also tabulated using Eq. (11).

**Table 5. Average number of events estimated to occur in Hong Kong and the probability of occurrence in a period of 50 years**

| Intensity $I$ | Return Period $T(I)$ of events with intensity greater than or equal to I estimated by Monte Carlo method (years) | Number of events occurred in 51 years (1918-40, 1954-81) | Average number of events estimated to occur in 50 years=$50/T(I)$ | Probability of occurring at least once in the next 50 years (%) |
|---|---|---|---|---|
| IV | 3.6 | 10 | 13.8 | 100 |
| V | 16 | 3 | 3.1 | 96 |
| VI | 38 | 1 | 1.3 | 74 |
| VII | 463 | 0 | 0.1 | 10 |

It is observed from Table 5 that there is a greater than 70% chance that another intensity VI tremor will occur during the next 50 years. Such a tremor corresponds to a ground acceleration of 0.03g. On the other hand, no intensity VII tremor has occurred during the past 51 years and the chance of getting one in the next 50 years is roughly one in ten. This is also the chance of having a ground acceleration due to earth tremors exceeding that required by the Hong Kong building code in the next 50 years.

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